Study of Coupling Optics Involving Hemispherical Microlens on the Tip of Single-Mode Step Index Fiber in the Framework of ABCD Matrix Which Takes Care of Arbitrary Angle of Incidence

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Abstract: We use the formulated ABCD matrix for refraction of Gaussian light beam by a spherical surface at arbitrary angle of incidence in order to study the launch optics involving laser diode to single-mode step index fiber coupling via hemispherical micro lens on the fiber tip. The reported investigation of coupling optics is realistic in the sense that it considers all possible angles of incidence. Further, to the best of our knowledge, no such study in case of easily fabricable hemispherical microlens on the fiber tip has been reported till date. Moreover, in our present investigation, we have taken into consideration the limited aperture provided by the hemispherical lens. The execution of this simple formalism, as regards estimation of the excitation efficiency, involves little computation and consequently it will be user friendly with the system engineers who are working in the field of optical technology.

Index Terms: Spot size; ABCD matrix; Laser diode; Hemispherical microlens; Single-mode circular core step index fiber; Coupling efficiency.

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1 Introduction

Microlenses are fabricated on the tip of the fiber in order to increase the laser diode to single mode fiber coupling efficiency [1-6].The microlenses are fabricated either in the conical or in the hemispherical shape. A microlens on fiber tip possesses the common advantage of being self-centered. Hyperbolic microlens on the fiber tip has large aperture so to collect the entire light emitted by laser diode and it is also free of spherical aberration and thus it emerges as the most efficient one in this context [3].Still, hemispherical microlens is used worldwide on account of the simplicity in its fabrication [3]. Numerous studies for optimum launch optics involving various types of microlens on the tip of mono mode fiber are available in literature [5-13]. It has been shown that the application of ABCD matrix formalism leads to prediction of the concerned coupling optics correctly but in a simple fashion [7-17]. The importance of graded index fiber is well known in view of its large bandwidth and insignificant sensitivity to macro as well as micro bending. This is why the study of graded index fiber in optimum launch optics is proliferating in literature. Very recently investigations on the coupling optics of hyperbolic microlens on graded index fiber tip have been reported [15,18]. The said

ABCD matrix formalism regarding estimation of relevant coupling optics is based on paraxial approximation.

Taking care of arbitrary angle of incidence, ABCD matrix for refraction by a spherical interface which separates media of refractive indices n_1 and n_2 respectively has been already reported [19,20]. Using this matrix, we evaluate coupling efficiency at different angles of incidence, compatible with the numerical aperture of the fiber, in case of laser diode to a single mode circular core step index fiber coupling via hemispherical microlens on the tip of the fiber. In order to make our prediction more realistic, we make the investigations for two wavelengths 1.3 μm and 1.5 μm [5]. The analysis involves optimization of the distance between the source and lens for maximum coupling efficiency corresponding to each angle of incidence for a given radius at a particular wavelength used. For the sake of simplicity and accuracy, we, employ Gaussian field distributions for both the source and the fiber as well (similar to previous workers [21,22] analysis also takes care of the limited aperture allowable by the hemispherical microlens [4]. Analytical expressions for the concerned coupling optics are prescribed and they are executable easily with little computation for the purpose of necessary estimations. This analysis will benefit the designers and engineers in the field of optical technology.

2 Theory

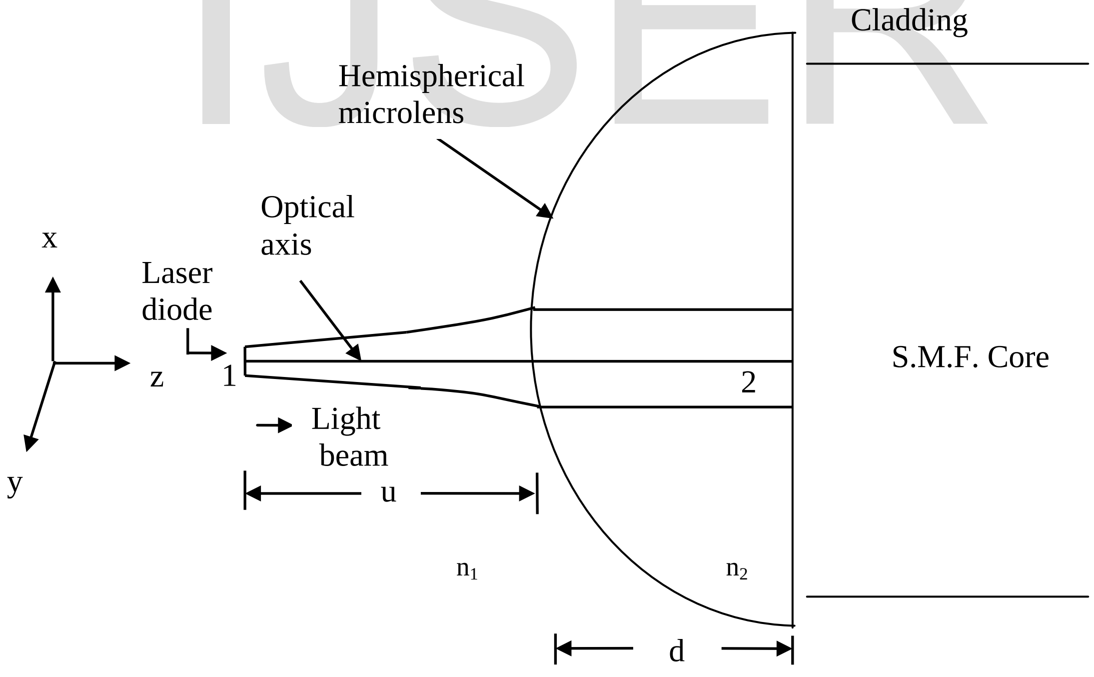

Fig.2: Schematic diagram of laser beam emitted from the input plane 1 of a laser diode and refracted through a hemispherical microlens to be incident on plane 2 which is the end face of a single-mode step index fiber

$$\psi_u = \exp\left[-\left(\frac{x^2}{w_{1x}^2}+\frac{y^2}{w_{1y}^2}\right)\right]\exp\left[-\frac{jk_1(x^2+y^2)}{2R_1}\right], \tag{1}$$

Here, R_1 denotes the radius of curvature of the incident wavefront while k_1 represents the wave number in the incident medium. It has been already verified that Gaussian approximations for the fundamental mode in the circular core single-mode fiber predict the relevant coupling optics excellently [5, 7-13]. Therefore, the fundamental mode in such a fiber can be taken as [5]

$$\psi_f = \exp\left[-\frac{(x^2+y^2)}{w_f^2}\right], \tag{2}$$

here, w_f stands for the spot size which, in case of step index fiber, can be approximated as

[23] :

$$w_f = a_{co}\left[0.65+\frac{1.619}{V^{1.5}}+\frac{2.879}{V^6}\right], \tag{3}$$

where, a_{co} is the core radius and V is the normalised frequency given by $V = k_0 a_{co}(n_{co}^2 - n_{cl}^2)^{\frac{1}{2}}$ with k_0 being wave number of free space and n_{co} and n_{cl} representing the refractive indices of the core and cladding respectively. Further, it is necessary that the polarised mode of the laser field should match with that of the circular core single-mode fiber.

The Laser field ψ_v on the fiber plane 2, transformed by the hemispherical microlens on the fiber tip, can be expressed as [14]:

$$\psi_v = \exp\left[-\left(\frac{x^2}{w_{2x}^2}+\frac{y^2}{w_{2y}^2}\right)\right]\exp\left[-\frac{jk_2}{2}\left(\frac{x^2}{R_{2x}}+\frac{y^2}{R_{2y}}\right)\right], \tag{4}$$

where, k_2 is the wavenumber in the lens medium while w_{2x}, w_{2y} are transformed spot sizes with R_{2x}, R_{2y} being radii of curvature of refracted wave front in the x and y directions respectively. In the appendix, we show the method of determination of $w_{2x,2y}$ and $R_{2x,2y}$ in terms of $w_{1x,1y}$ and R_1 by ABCD matrix formalism [14,19]. Further, without consideration for finite aperture of hemispherical lens the source to single-mode fiber coupling efficiency (η_0) with hemispherical microlens on the tip of the fiber is evaluated by using the overlap integral given below [7-18]:

$$\eta_0 = \frac{\left|\int\int \psi_v \psi_f^* dxdy\right|^2}{\left|\int\int|\psi_v|^2 \, dxdy \int\int|\psi_f|^2 \, dxdy\right|}, \qquad (5)$$

Using Eqs. (2) and (4) in Eq.(5), we get:

$$\eta_0 = \frac{4w_{2x}w_{2y}w_f^2}{\left(\left[(w_f^2+w_{2x}^2)^2+\frac{(k_2^2w_f^4w_{2x}^4)}{(4R_{2x}^2)}\right]^{\frac{1}{2}}\left[(w_f^2+w_{2y}^2)^2+\frac{(k_2^2w_f^4w_{2y}^4)}{(4R_{2y}^2)}\right]^{\frac{1}{2}}\right)}, \qquad (26)$$

The coupling efficiency of a hemispherical lens is lowered owing to its limited aperture provided for transmission of optical beam through it. The radius ρ_c above which transmission is not allowed by a hemispherical lens of radius a has been found as [4]:

$$\rho_c = \frac{n_1 a}{n_2}, \qquad (7)$$

where, n_1 and n_2 present refractive indices of incident and lens media respectively. Accordingly lens transmitivity factor T is given by [4]:

$$T = \frac{\int_0^{\rho_c}|\psi_f t|^2 \, rdr}{\int_0^{\infty}|\psi_f|^2 \, rdr}, \qquad (8)$$

Further, the transmission coefficient t of the material of lens is approximated as [4]:

$$t = \frac{2(n_1 n_2)^{\frac{1}{2}}}{n_1 + n_2} \qquad (9)$$

Employing Eqs. (2), (7) and (8), we get:

$$T = t^2\left[1-\exp\left(-\frac{2\rho_c^2}{w_f^2}\right)\right], \qquad (10)$$

Finally, the actual coupling efficiency (η) will be given by,

$$\eta = \eta_0 \, T, \qquad (11)$$

3 Results and discussions

We employ the formulated matrix for study of coupling of laser diode with single-mode step index fiber via hemispherical microlens on the fiber tip at different angles of incidence. In this connection, we restrict our analysis to two relevant wavelengths namely $\lambda = 1.5\mu m$ (w_1x=0.843μm, w_1y=0.857μm) and 1.3μm (w_1x =1.081μm, w_1y =1.161μm) [5]. Further, in each case, the spot size (w_f) of the fiber is 4.794μm while the refractive index of the material of microlens with respect to the outside medium is taken as1.55 [5]. Here, we also take the refractive indices of core and cladding as 1.461 and 1.455 respectively and this gives semi vertical angle of the acceptance cone of light around 7.5^0. Again, it has been found that prediction of coupling optics based on planar wavefront model for the input beam from the laser facet is almost identical with that based on spherical wavefront model[5,7,8]. Accordingly in our present study, we employ planar wavefront model for the input beam for the sake of simplicity and accuracy as well. In this connection, we choose hemispherical microlens of radii 3μm, 4μm, 5μm, 6μm, 7μm and 8μm. For a given radius at a particular wavelength, we evaluate the maximum coupling efficiency for each angle of incidence, the maximum angle of study being semi vertical angle of acceptance cone for obvious reasons. It is relevant to mention in this connection that in each case for each angle of incidence, the distance between laser diode and lensed fiber has to be optimized in order to get maximum coupling efficiency. It is to be noted that the coupling efficiency under paraxial approximation corresponds to $\theta = 0^0$and we have ready reference to this in each graph. In Figs. 3a,4a,5a,6a,7a and 8a we have plotted the maximum coupling efficiency versus angle of incidence for lens radii being 3μm ,4μm , 5μm, 6μm, 7μm and 8μm respectively for the wavelength1.3μm,while Figs.

3b,4b,5b,6b,7b and 8b depict the same behaviour at wavelength 1.5μm. It is found that the coupling efficiencies calculated taking care of different angles of incidence differ from those found on the basis of paraxial approximation by small amount. It is seen from Figs. 3(a) and 3(b) that for a=3μm, η =16.3376% for $\theta_1 = 0^0$ at λ= 1.3μm and η =26.4053% for $\theta_1 = 0^0$ at λ= 1.5μm and moreover in each case efficiency increases with increase of angle of incidence. It is seen from Figs. 4(a) and 4(b) that for a = 4μm, η = 35.6134% for $\theta_1 = 0^0$ at λ= 1.3μm and η = 47.8856% for $\theta_1 = 0^0$ at λ= 1.5μm and for each wavelength efficiency increases with increase of angle of incidence. It is seen from Figs. 5(a) and 5(b) that for a = 5μm, η = 56.5585% for θ1 = 0^0 at λ= 1.3μm and η =64.1691% for $\theta_1 = 0^0$ at λ= 1.5μm and at λ= 1.3μm efficiency increases with increase of angle of incidence while at λ= 1.5 μm, efficiency decreases with increase of angle of incidence. It is seen from Figs. 6(a) and 6(b) that for a = 6μm, η =73.5947% for $\theta_1 = 0^0$ at λ= 1.3μm and η = 72.1102% for $\theta_1 = 0^0$ at λ= 1.5μm and at λ= 1.3μm, efficiency increases with increase of angle of incidence while at λ= 981.5μm, efficiency decreases with increase of angle of incidence. It is seen from Figs. 7(a) and 7(b) that for a = 7μm, η =84.1487% for $\theta_1 = 0^0$ at λ= 1.3μm and η = 72.9730% for $\theta_1 = 0^0$ at λ= 1.5μm and at λ= 1.3μm efficiency increases with increase of angle of incidence while at λ= 1.5 μm , efficiency decreases with increase of angle of incidence. It is seen from Figs. 8(a) and 8(b) that for a = 8μm, η = 88.4875% for $\theta_1 = 0^0$ at λ= 1.3μm and η =69.4410% for $\theta_1 = 0^0$ at λ= 1.5μm and at both the wavelengths, efficiency decreases with increase of angle of incidence. Bearing in mind the constructional difficulties involving hemispherical lens of radius more than 6μm, we have carried on our investigation up to lens radius 8μm with the hope that improvement of technology in future may make construction of hemispherical microlens of radius more than 6μm possible.

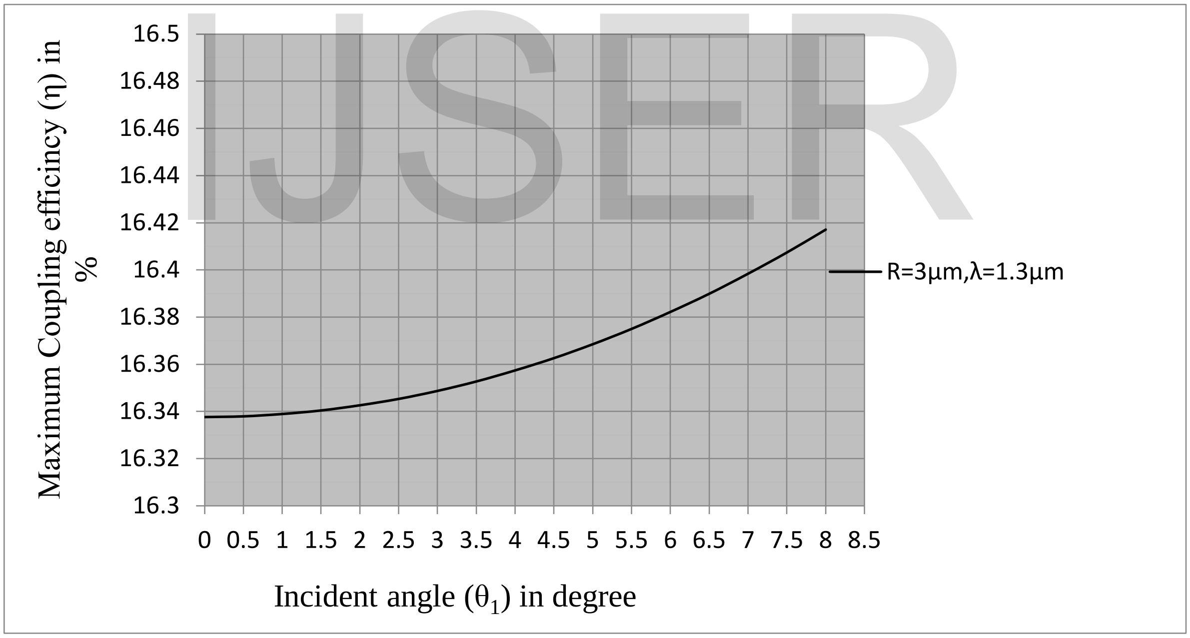

Fig.3a: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

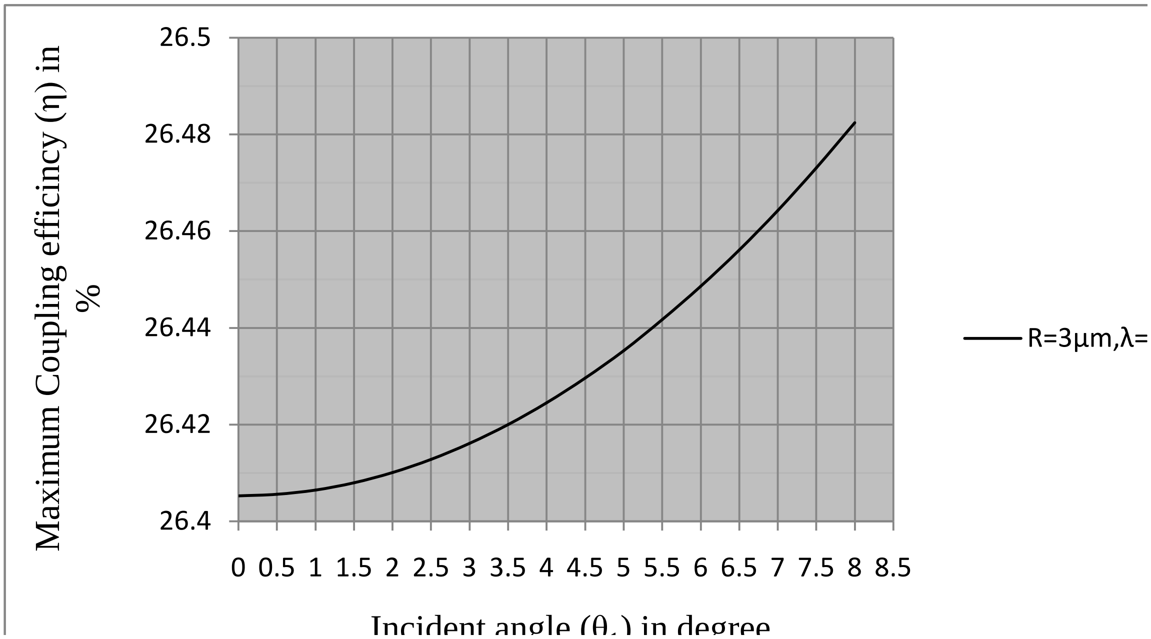

Fig.3b: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

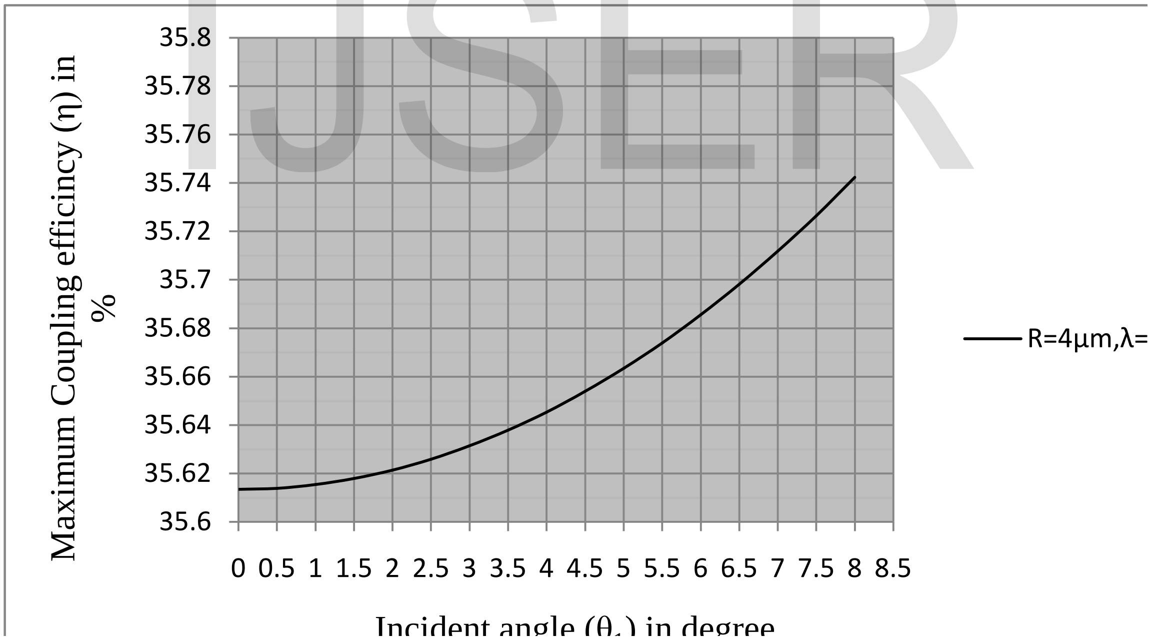

Fig.4a: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

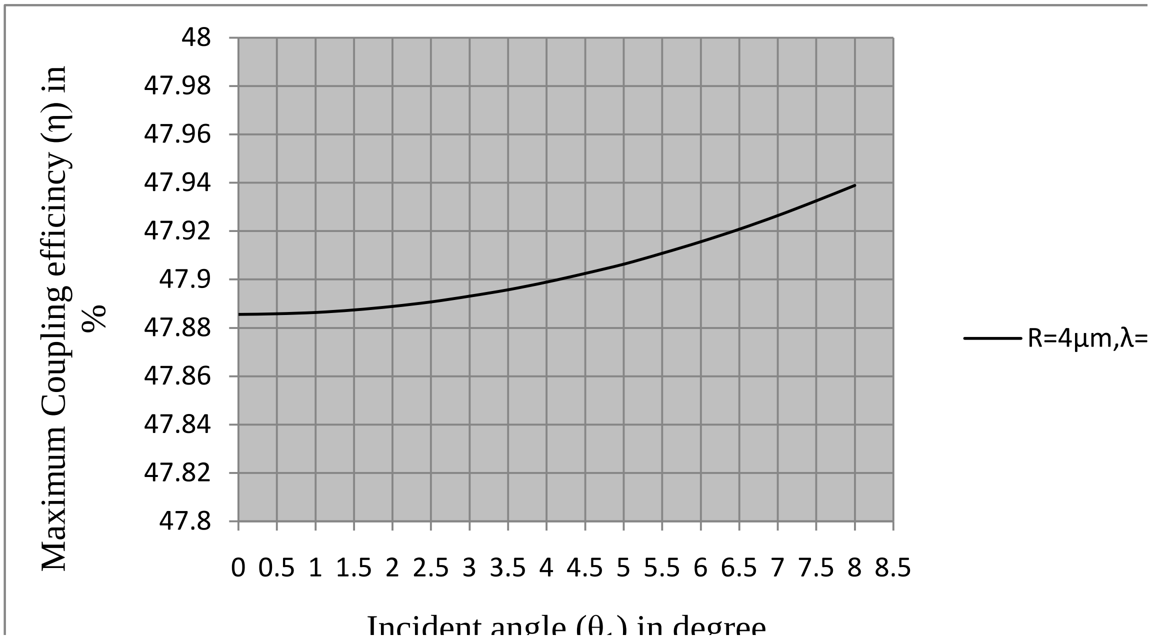

Fig.4b: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

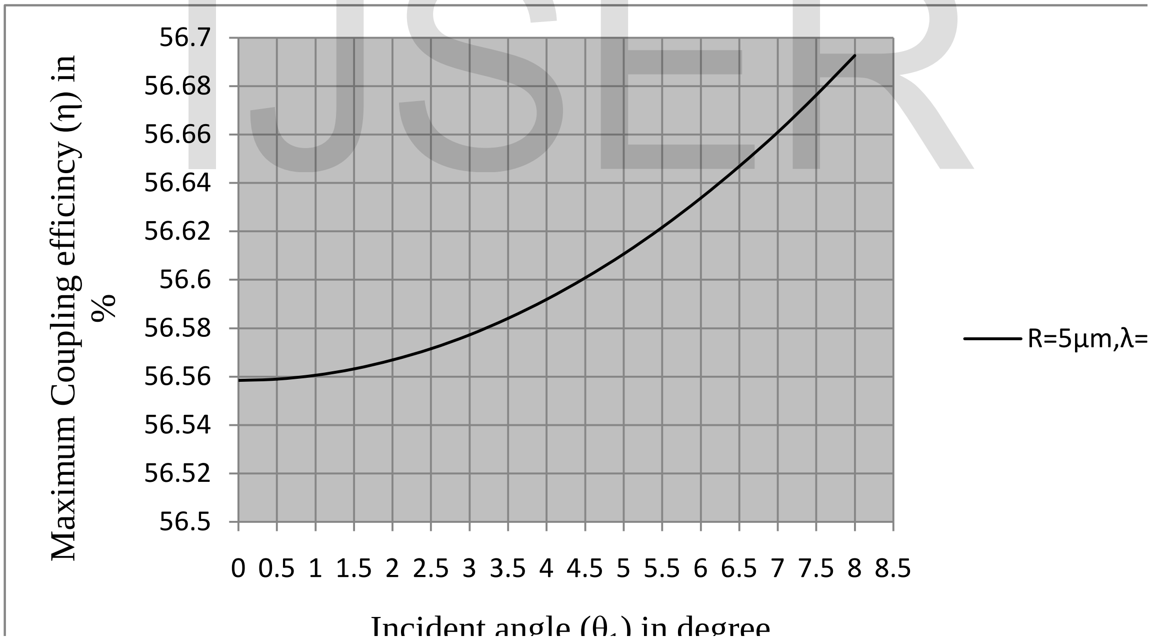

Fig.5a: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

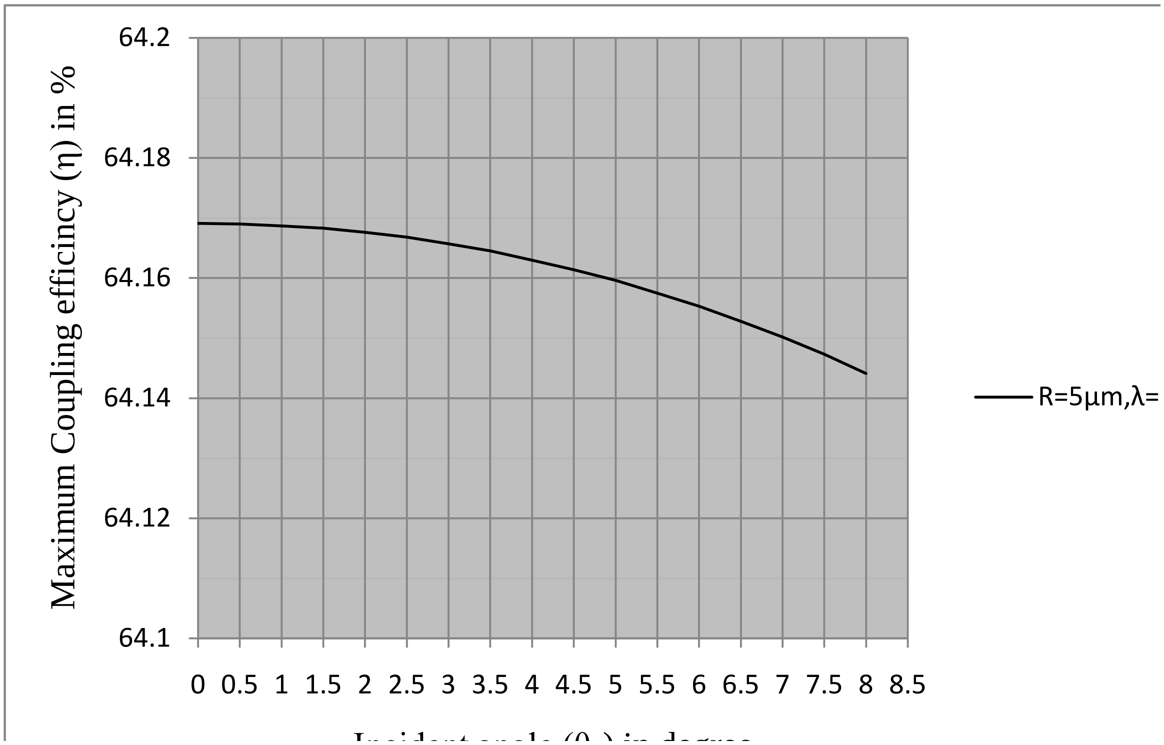

Fig.5b: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

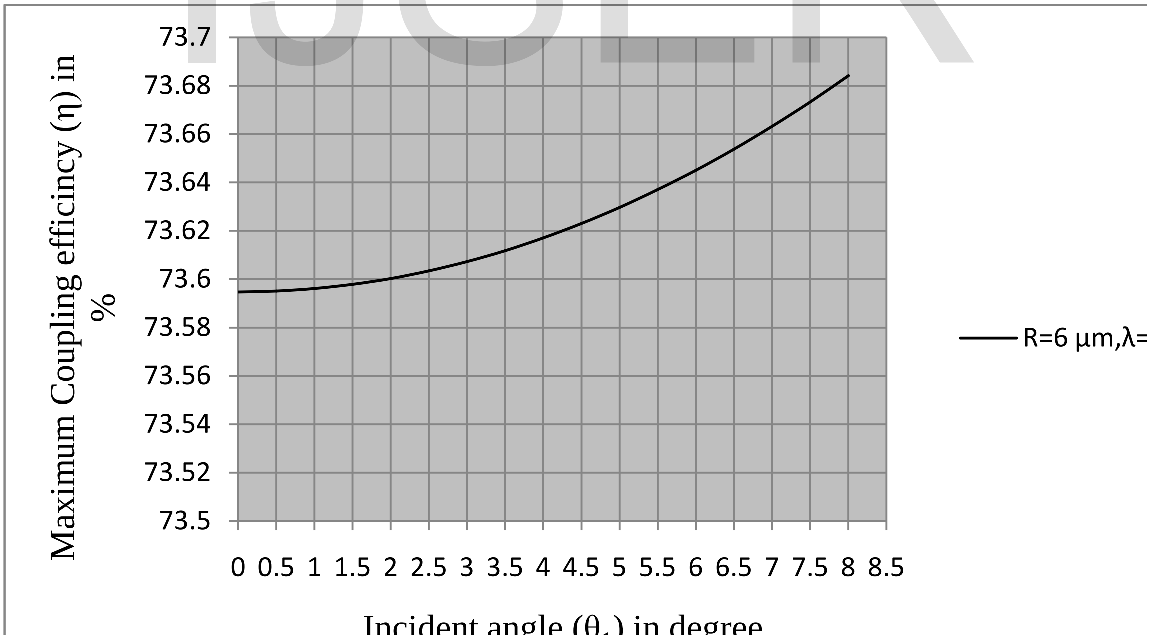

Fig.6a: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

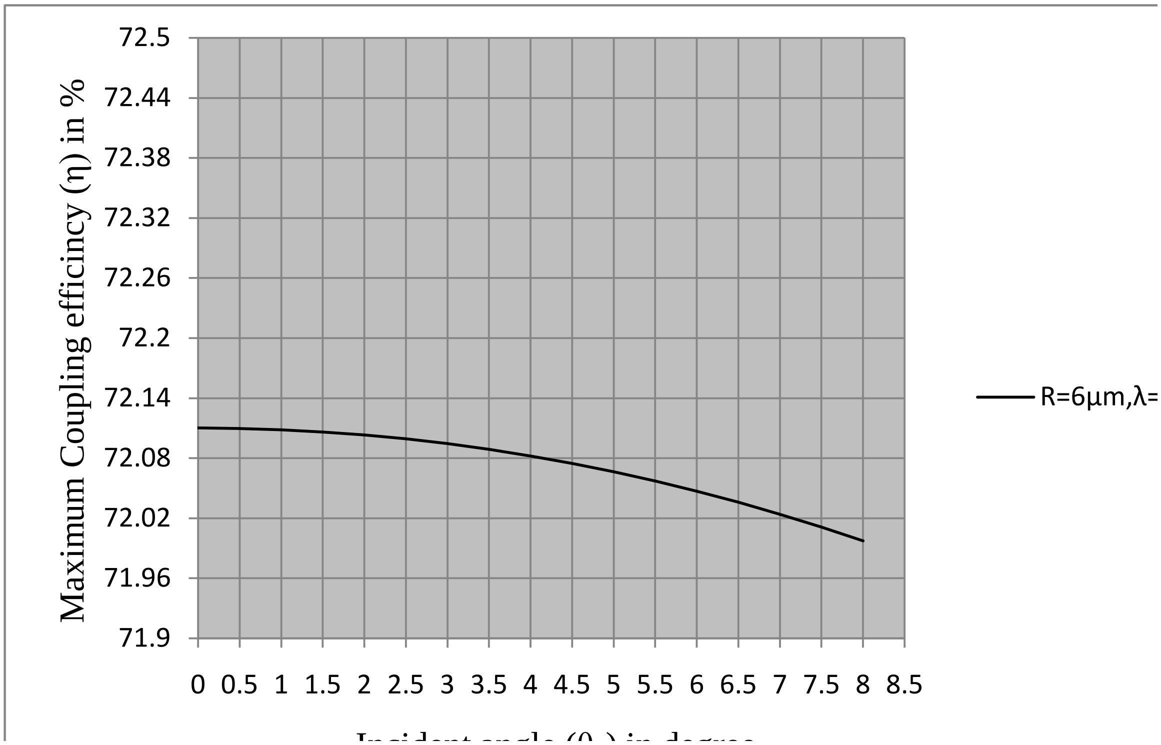

Fig.6b: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

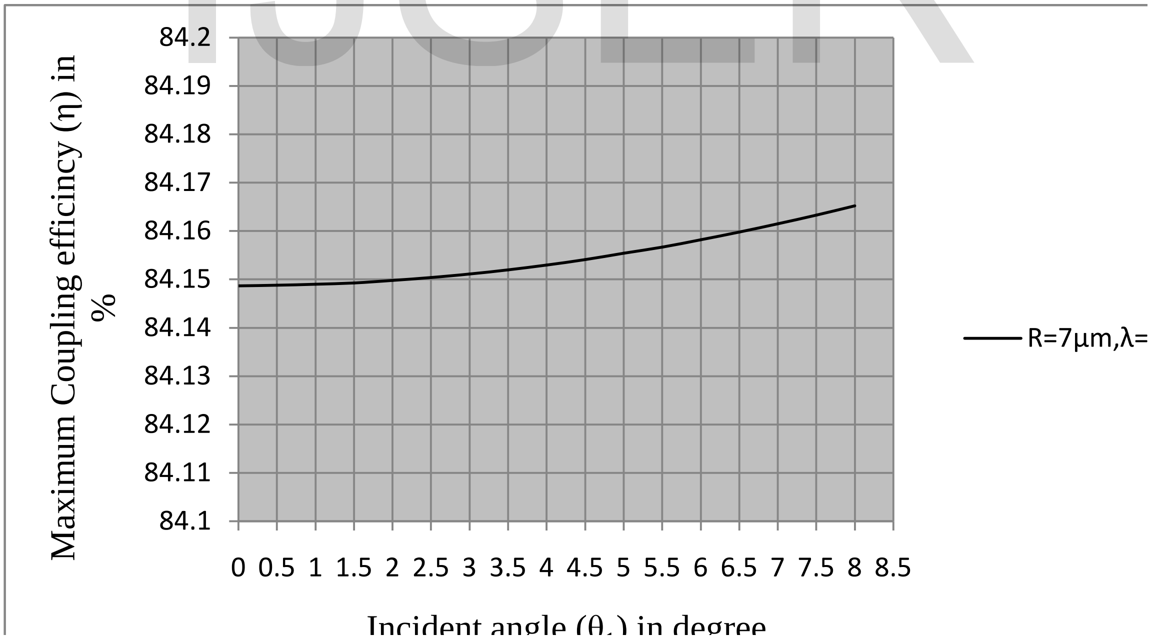

Fig.7a: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

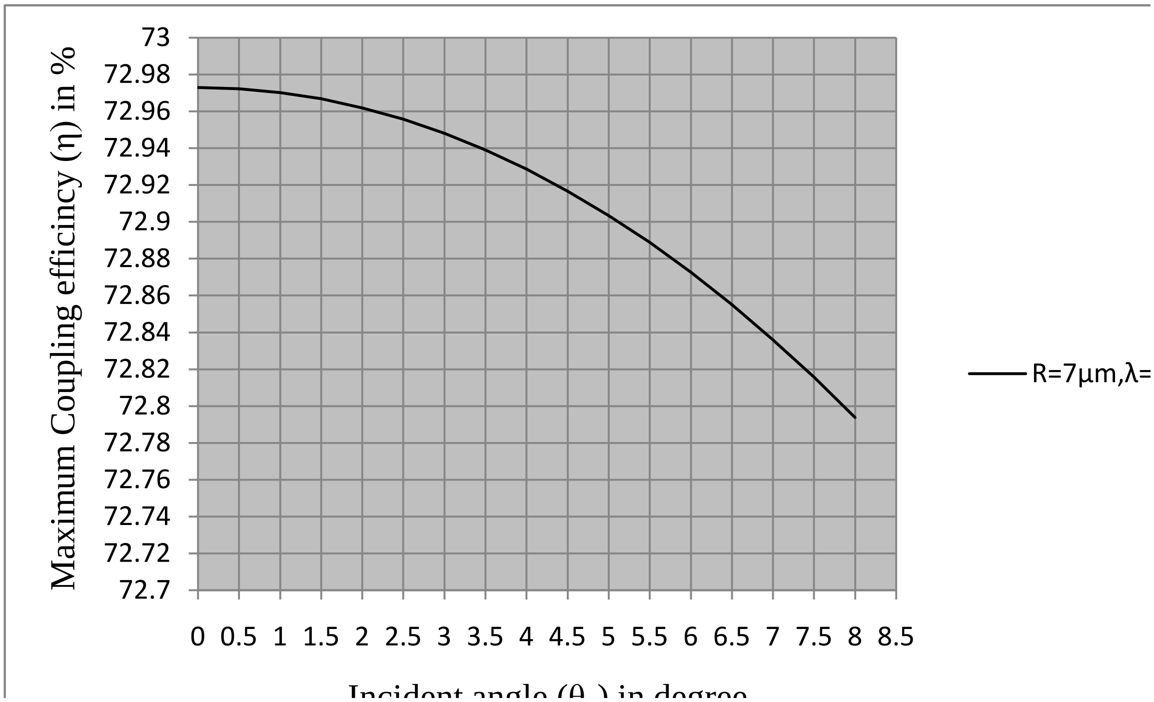

Fig.7b: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

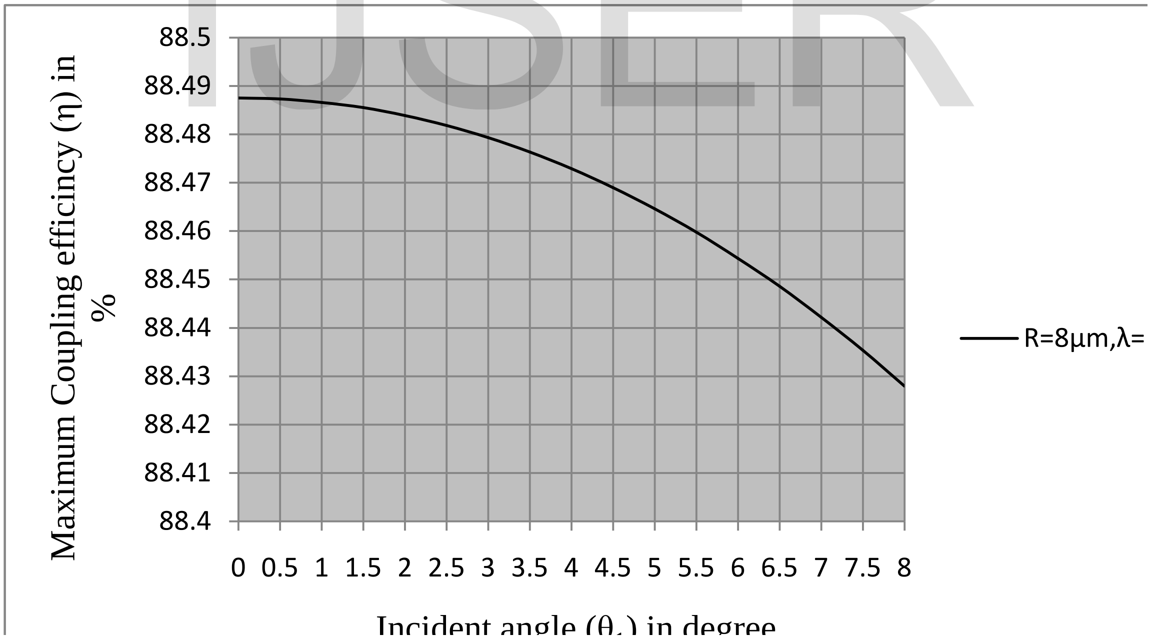

Fig.8a: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

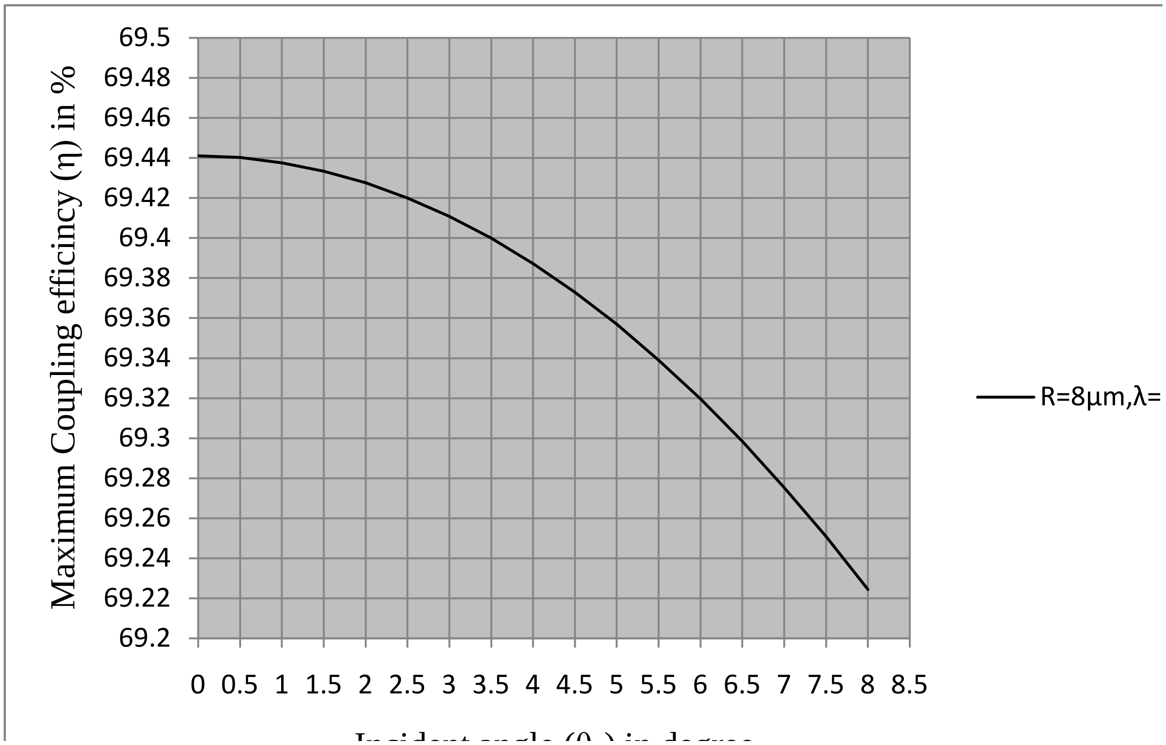

Fig.8b: Variation of maximum coupling efficiency (η) with angle of incidence (θ_1)

4 **Conclusion**

Conclusively, the ABCD matrix in case of refraction of Gaussian laser beam at arbitrary angle of incidence from a spherical interface has been prescribed has been used here to evaluate the laser diode to single-mode step index fiber coupling efficiency at different angles of incidence in presence of hemispherical microlens on the fiber tip. The application of ABCD matrix formalism has simplified calculations to a large extent. The excellent agreement of our prediction with those for paraxial rays available in literature justifies the correctness of our technique. Accordingly, this user friendly formalism will benefit the designers and packagers working in the field of optimum launch optics.

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Appendix

The relation between output parameter q_2 and input parameter q_1 is given by the following expression:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \quad , \tag{A1}$$

where

$$\frac{1}{q_{1,2}} = \frac{1}{R_{1,2}} - \frac{j\lambda_0}{\pi n_{1,2} w_{1,2}{}^2} \quad , \tag{A2}$$

The ray matrix for refraction by the hemispherical microlens of radius *a* on the fiber tip can be expressed as [19]:

$$\begin{aligned} M &= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \\ &= \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{1 - n\cos(\theta_1 - \theta_2)}{na} & \dfrac{1}{n} \end{pmatrix} \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix} \end{aligned} \tag{A3}$$

where,

$$\begin{aligned} A &= 1 + \frac{a\{1 - \cos(\theta_1 - \theta_2)\}}{na}, \\ B &= u + \frac{u\,a\{1 - \cos(\theta_1 - \theta_2)\}}{n\,a} + \frac{a}{n}, \\ C &= \frac{\{1 - \cos(\theta_1 - \theta_2)\}}{n\,a}, \\ D &= \frac{1}{n} + \frac{\{1 - \cos(\theta_1 - \theta_2)\}u}{n\,a} \end{aligned} \tag{A4}$$

Here, *u* is the distance of the laser diode from the hemispherical microlens and the maximum depth d of the lens is equal to its radius of curvature *a*. The refractive index of the material of the lens with respect to outside medium is denoted by *n* where $n = \frac{n_2}{n_1}$.

The lens transformed spot sizes $w_{2x,2y}$ and radii of curvature $R_{2x,2y}$ are found by using Eq. (A4) in Eqs. (A1) and (A2) and those are given below:

$$w_{2x,2y}^{2}=\frac{A_1^2 w_{1x,1y}^2+(\lambda_1^2 B^2)/w_{1x,1y}^2}{n\left(A_1 D-BC_1\right)}, \tag{A5}$$

$$\frac{1}{R_{2x,2y}}=\frac{A_1 C_1 w_{1x,1y}^2+\left(\lambda_1^2 BD\right)/w_{1x,1y}^2}{A_1^2 w_{1x,1y}^2+\left(\lambda_1^2 B^2\right)/w_{1x,1y}^2}, \tag{A6}$$

where, $\lambda_1=\frac{\lambda}{\pi},\ \lambda=\frac{\lambda_0}{n_1},\ A_1=A+\frac{B}{R_1}\ and\ C_1=C+\frac{D}{R_1}$ and λ_0 is the wavelength of light in free space.

References

[1] J. H. Song, H.N. J. Fernando, B. Roycroft, B. Corbett , F. H. Peters, Practical design of lensed fibers for semiconductor laser packaging using laser welding technique, J. Lightwave Technol. **27** (2009),1533-1539.

[2] E. Li, Characterization of fiber lens, Opt. Lett. **31** (2006), 169-171.

[3] H. M. Presby, C. A. Edwards, Near 100% efficient fibre microlenses, Electron Lett. **28** (1992), 582-584.

[4] C. A. Edwards, H. M. Presby , C. Dragone, Ideal miccrolenses for laser to fiber coupling, J. Lightwave Technol. **11** (1993)**,** 252-257.

[5] J. John, T. S. M. Maclean, H. Ghafouri-Shiraz , J. Niblett, Matching of single-mode fibre to laser diode by microlenses at 1.5 -1.3 μm wavelength, IEE Proc.Optoelectron. **141**(1994), 178-184.

[6] H. M. Presby, C. A. Edwards, Efficient coupling of polarization maintaining fiber to laser diodes,IEEE Photon. Technol. Lett. **4** (1992), 897-899.

[7] S. Gangopadhyay , S. N. Sarkar, Laser diode to single-mode fibre excited via hyperbolic lens on the fibre tip: formulation of ABCD matrix and efficiency computation, Opt. Commun.**132** (1996), 55-60.

[8] S. Gangopadhyay , S. N. Sarkar, ABCD matrix for reflection and refraction of Gaussian light beams at surfaces of hyperboloid of revolution and efficiency computation for laser diode to single-mode fiber coupling by way of a hyperbolic lens on the fiber tip, Appl. Opt.**36** (1997), 8582-8586.

[9] S. Mukhopadhyay, S. Gangopadhyay , S. N. Sarkar, Coupling of a laser diode to a monomode elliptic-core fiber via a hyperbolic microlens on the fiber tip: efficiency computation with the ABCD matrix, Opt. Engg. **46** (2007), 1-5.

[10] S. Mukhopadhyay , S. N. Sarkar, Coupling of a laser diode to single mode circular core graded index fiber via hyperbolic microlens on the fiber tip and identification of the suitable refractive index profile with consideration for possible misalignments, Opt. Engg. **50** (2011),1-9.

[11] S.Mukhopadhyay , S. Gangopadhyay , S.N. Sarkar , Misalignment consideration in a laser diode to monomode elliptic core fiber coupling via a hyperbolic microlens on the fiber tip: efficiency computation by the ABCD matrix, Opt. Engg. **46** (2007), 1-5.

[12] S.K. Mondal , S.N. Sarkar , Coupling of a Laser Diode to Single-Mode Fiber with an Upside -Down Tapered Lens End, Appl. Opt. **38** (1999),6272-6277.

[13] S.Gangopadhyay , S.N. Sarkar, Misalignment Considerations in Laser Diode to Single-Mode Fibre Excitation Via Hyperbolic Lens on the Fibre Tip,Opt. Commun.**146** (1998), 104-108.

[14] S. Gangopadhyay , S. N. Sarkar, Laser diode to single-mode fibre excitation via hemispherical lens on the fibre tip: efficiency computation by ABCD matrix with consideration for allowable aperture, J. Opt. Commun. **19** (1998), 42-44 **.**

[15] B.Das, A.K. Maiti, S. Gangopadhyay, Laser diode to single-mode circular core dispersion-shifted/dispersion- flattened fiber excitation via hyperbolic microlens on the fiber tip: Prediction of coupling efficiency by ABCD matrix formalism, Optik 125(2014) 3277–3282.

[16] B.Das, T.R. Middya, S. Gangopadhyay, Mismatch considerations in excitation of Single-mode Circular Core Parabolic Index Fiber by Laser Diode via Upside Down Tapered Hemispherical Microlens on the Tip of the Fiber, J. Opt. Commun. DOI: 10.1515/joc-2016-0043.

[17] Shubhendu Maiti, Anup Kumar Maiti, Sankar Gangopadhyay,Laser diode to single-mode triangular-index fiber excitation via upside down hemispherical microlens on the fiber tip: Prescription of ABCD matrix of transmission and estimation of coupling efficiency, Optik **144** (2017) 481-489.

[18] Angshuman Majumdar, Chintan kumar Mandal and Sankar Gangopadhyay, Laser diode to single-mode circular core parabolic index fiber coupling via Upside Down Tapered Hyperbolic micro lens on the tip of the fiber: Prediction of coupling optics by ABCD matrix formalism. Journal of Optical Communications (Germany) DOI:10.1515/JOC-2017-0040

[19] Shubhendu Maiti, Anup Kumar Maiti,Formulation of ABCD matrix for reflection and refraction of Gaussian light beams on the hemispherical microlens drawn on the tip of fiber, International Journal of Computer Applications (0975-8887)

[20] A. Yariv, Optical Electronics, Saunders College Publishing (1991).

[21] S. Sarkar, K. Thyagrajan , A. Kumar, Gaussian approximation of the fundamental mode in single mode elliptic core fibers, Opt. Commun. **49** (1984), 178-183.

[22] D. Marcuse, Gaussian approximation of the fundamental modes of graded index fibers, J. Opt. Soc. Am. **68** (1978), 103-109.

[23] D. Marcuse, Loss analysis of single-mode fiber splices, Bell Syst. Tech. J. **56** (1977), 703-718.

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